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PRINCIPLES OF MATHEMATICAL SIMULATION OF MINING VEHICLES USING APPLIED MATHEMATICAL SOFTWARE

Abstract. The transportation of rock mass and coal in mining operations relies heavily on the efficiency and reliability of mining transport systems. This study delves into the intricate dynamics of chassis functions during transportation, emphasizing the critical role of transmitting dynamic stresses and mass to the road or rail structures. With a focus on increasing productivity, modern mining locomotives and heavy dump trucks now carry substantial adhesion adhesive weights, allowing for the hauling of heavier loads even on steep slopes.

The integration of scientific simulations and research-based design methods aids in identifying and managing dynamic loading effects within mining vehicle chassis. This approach minimizes the transfer of dynamic loads onto the bolster structure, enhancing system reliability and operational efficiency. Computer-aided software plays a vital role in streamlining the development process, reducing the need for costly physical prototypes and extensive testing in challenging mining environments.

The study outlines simplified calculation schemes for mining transport utilities, balancing the need for accuracy with computational efficiency. By utilizing mathematical models and simulation techniques, the dynamics of mine locomotives and dump trucks can be accurately evaluated, guiding design decisions and operational strategies. The application of Lagrange equations and software tools like "Wolfram Mathematica" facilitates the generation of differential equations for dynamic analysis, providing insights into vehicle dynamics and road interactions.

Overall, this research underscores the importance of advanced modeling and simulation techniques in optimizing mining transport systems, enhancing safety, productivity, and reliability in demanding mining environments.

Purpose. To highlight the application of mathematical software as an instrument for mechanical system dynamics study, that allows scientifically substantiate the usage of new technical solutions during their development.

Method. Development a system of differential equations using Lagrange equations of the 2nd order, which are solved in Wolfram Mathematica. Preparation of initial conditions and mass-inertia data can be done by any known software. The generalized mathematical model can be used for any vehicle suspending unnecessary coordinates.

Results. Implementation of new technical solutions without scientific ground could be resulted in difficulties while exploitation. For its removal necessary to receive dynamical characteristics. Existing engineering calculation methods are not suitable, especially because of tight time and limited financing. The customized mathematical models can be developed and solved on demand using available software.

A scientific novelty. Enhancement of theoretical and experimental research become possible owing to the usage of modern approaches of dynamic systems simulation using applied mathematical software Wolfram Mathematica. Obtained model can be modified in few operation from one mass task into multibody system with maximum 69 free displacements (coordinates).

Practical value. The possibility to receive customized simulation model fast, verified with increased quality for scientific purposes.

Keywords: dynamics, simulation, rail-wheel interaction, dynamical stress, mining rolling stock research.

Introduction

During the rock mass and coal transportation by the mining transport along the mining shafts, the chassis' functions are not only carrying static loads, but to transmit the dynamical stress and frame mass to the road or rail track structure as well. The interaction area between wheel and track ensures transmitting braking and tractive forces. In order to increase the productivity of the mining rolling stock, an adhesive weight of the modern mining locomotives increases either and now achieves 10-28 tons and up to 450 tons for heavy dump trucks. This mass allows hauling heavier cargo with significantly increased static loads on chassis on the steeper slopes. Due to the fact, that existing mining transport requires to meet special safety regulation, comprehensive research on strength of each link must be provided on the development stage.

Each of mining drifts has its own climate environment, road profile and plan, bending radii, track incline, admissible haulage speed

and braking distance etc. All these factors influence on both economic and exploitation indexes, and on transport system reliability in general. Thus, mathematical simulation of the transport unit is actual scientific and engineering challenge.

Modern design methods, which base on the scientific simulation and research approaches, facilitate definition of the location and character of arising dynamical loading and prevent their growth during forming within the mining vehicle chassis.

This prevents the following dynamical load transfer on the bolster structure. Thus, the structure selection and selection of mining machines parameters, which bases on the detailed analysis of running processes, might be an essential part of energy-mechanical system and its scheme development.

As it is known, significant meaning for dynamical systems which are at issue, is provided by their design features, and first of all coupling characters between separated parts and units of vehicle.

The usage of computer-aided software while solving assigned tasks should reduce working hours during development a new system of mining rolling stock avoiding lasting produce stage of expensive models, pilot units, their tests, especially in mining conditions. This synthesis section is an integral part of sophisticated energy-mechanical system and must be integrated into development procedure on the designing stage. Thus, it is actual issue of development and exploitation.

Previous research analysis

Calculation scheme of each mining transport utility (**Fig. 1**) can include some simplifications, which do not significantly influence on design characteristics (wheel rigidity, clearances, type of ballast {these parameters are unlikely to be considered as dynamical as they are not depend on the speed}). Masses of a frame, chassis, wheel and transmission are reduced to one mass. Also authors propose to simulate both rail locomotive wheelset and pneumatic wheel in the same way but with different rigidity, that simplifies the mathematical model and increases the efficiency. In general case, coupling of wheel and chassis is unilateral and non-holonomic both in vertical and horizontal (lateral slip exists) surfaces. In this case, calculation scheme can be correctly described by Lagrange equation. But if the whole system is divided on two separate parts (one for unsprung mass, another sprung mass) two mathematical models, connected by edges of road-wheel interaction, could be generated. To solve one of the separated models influence might be substituted with corresponding road (rail) reactions, which influence on chassis' wheel using D'Alembert principle.

The research results

The calculation scheme of vehicle generally can be described by 69 independent coordinates for longitudinal x, lateral y and vertical z wheel and chassis displacements (for locomotives); frame's angular oscillations – yaw ψ, roll θ and pitch φ. Torsional oscillation on transmission members φ are considered as well as their angular rigidity c. All denoted locomotives elements have masses m and inertia moment I. The couplings, which characterizes by rigidity, have energy dissipation and marked as dissipation coefficient β.Calculation scheme of mine locomotive (**Fig. 1**) consists some simplifications, which do not significantly influence on design characteristics (wheel rigidity, clearances, type of ballast). Masses of a frame, chassis, wheel-pairs and transmission are reduced to one mass. However, the model with 69 independent coordinates requires generating of more than 150 equations. Therefore, while simulation of simplified parts of the vehicle there is no necessity to generate the whole model. Only a part of required coordinates is sufficient.

The example of such simplified model is a mathematical model of mining dump truck (**Fig. 2**), with one unsprung mass. The wheels are simulated as an independent elastic-dissipative links.
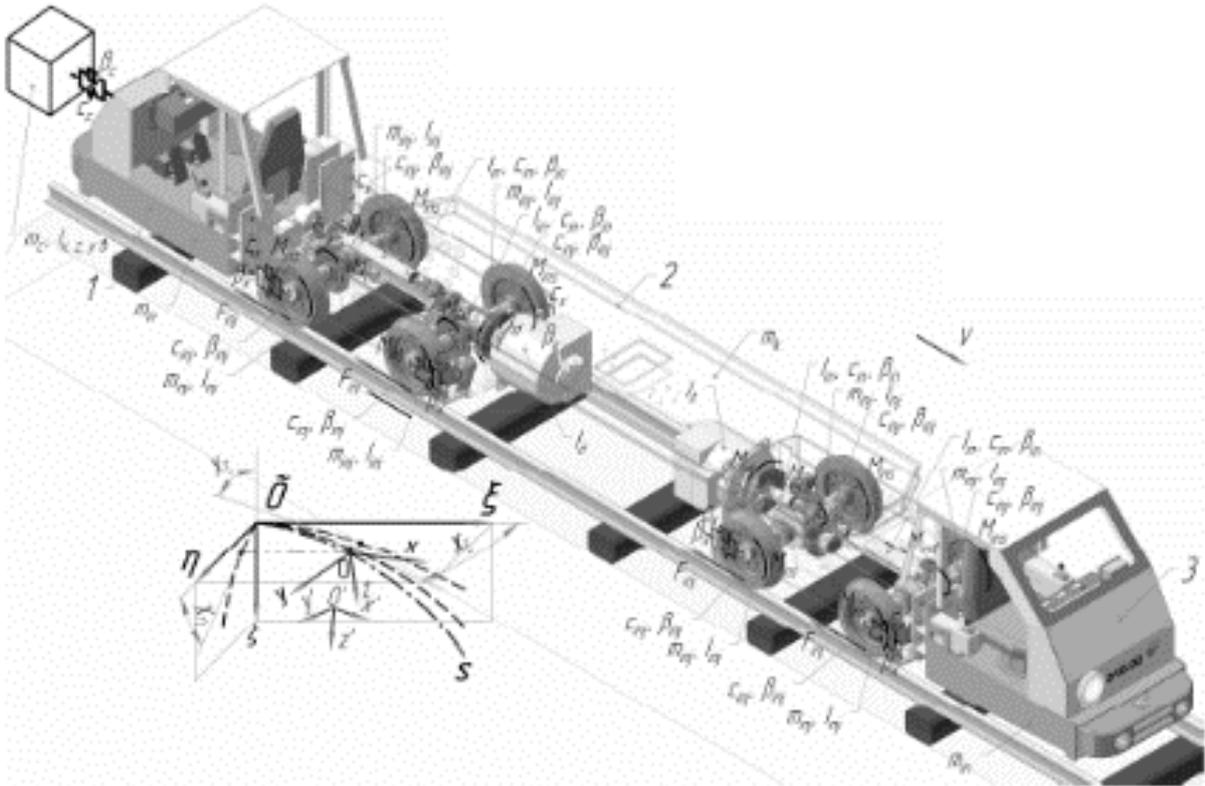
Fig. 1. Calculation scheme of mine locomotive

In case of simulation of mining dump truck the same variables can be used but unnecessary coordinates can be suspended.

For example of such mathematical model a dump truck calculation scheme is provided on the **Fig. 2**.
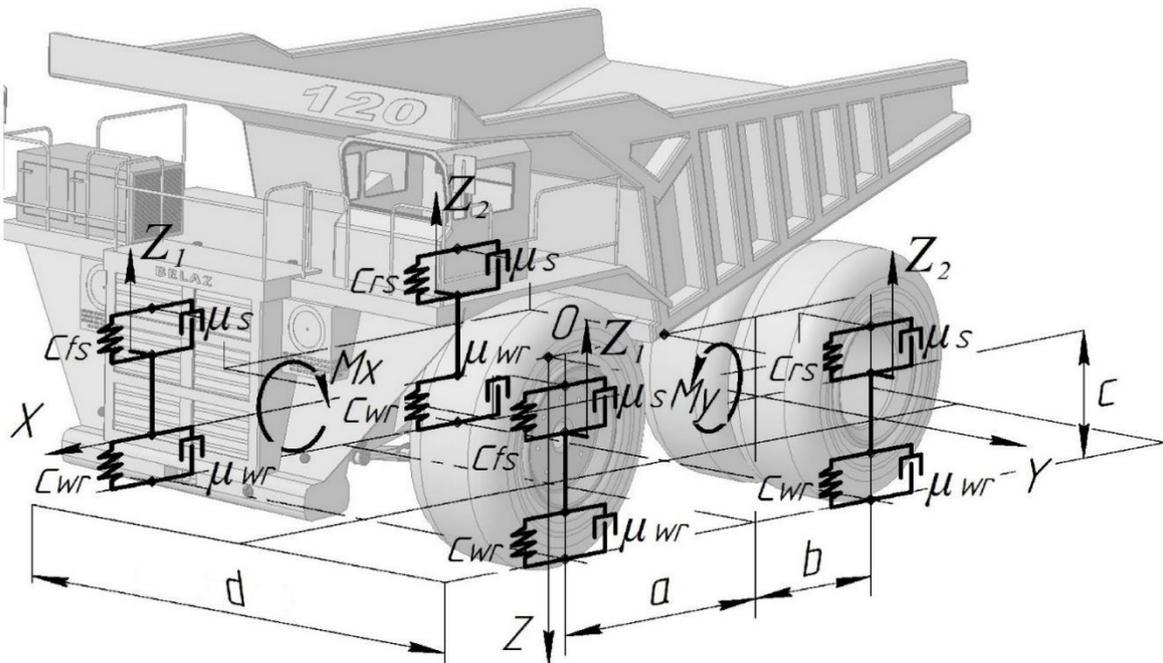
Fig. 2. Calculation scheme of dump truck

To compile the equations of motion we use the Lagrange equation of the second order, where in the preparation of expressions for the kinetic and potential energy of T, П, the dissipation function D, we used the values of mass and moments of inertia of m1, m2, mV,

JYB, JXB, JY1, JY2. As generalized coordinates Qi were chosen X, Z1, Z2, Z, φ1, φ2, φ, ψ). Three components can be described as given below:

The kinetic energy of the system:

$$T = \frac{1}{2} \cdot (J_{XB} \cdot \dot{\psi}^2 + J_{YB} \cdot \dot{\varphi}^2 + \dot{X}^2 \, (\frac{2(J_1 + J_2)}{r^2} 2m_1 +$$

$$+ 2m_2 + m_B) + J_{Y1} \cdot \dot{\varphi}_1^2 + J_{Y2} \cdot \dot{\varphi}_2^2 + m_B \cdot \dot{Z}^2 +$$

$$+ m_1 \cdot \dot{Z}_1^2 + m_2 \cdot \dot{Z}_2^2) \tag{1}$$

The potential energy of the system (2):

$$\Pi = \frac{1}{2} \cdot (c_s (\Delta_1^2 + \Delta_2^2 + \Delta_3^2 + \Delta_4^2) +$$

$$+ c_{wr} (\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2)) \tag{2}$$

The dissipation function (3):

$$D = \frac{1}{2} \cdot (\mu_s (\dot{\Delta}_1^2 + \dot{\Delta}_2^2 + \dot{\Delta}_3^2 + \dot{\Delta}_4^2) +$$

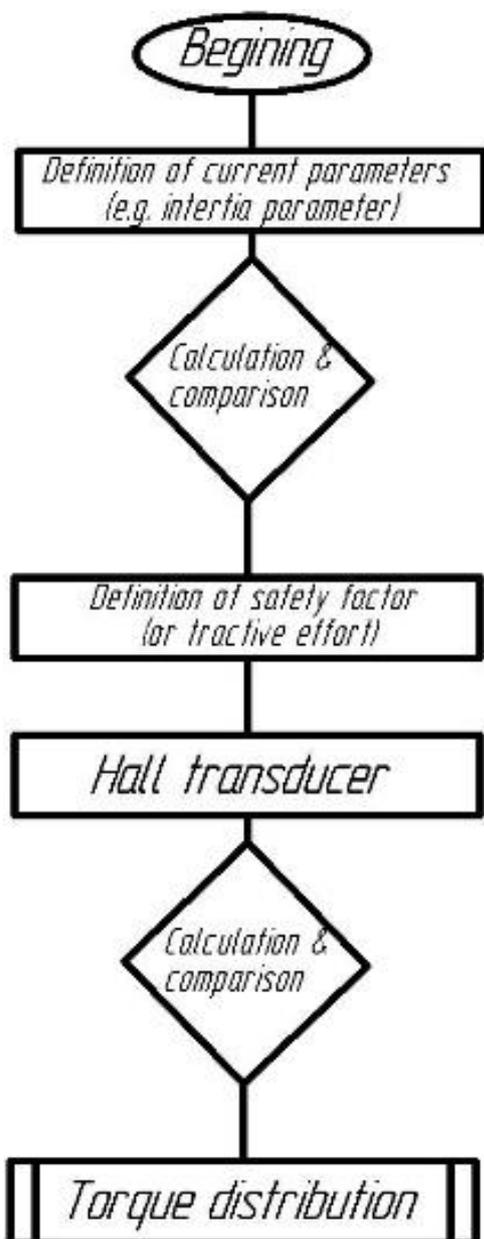$$+ \mu_{wr} (\dot{\delta}_1^2 + \dot{\delta}_2^2 + \dot{\delta}_3^2 + \dot{\delta}_4^2)) 3 \tag{3}$$

where Δi, i – respectively displacement and velocity of the center of mass body dump, in the motion for a given profile path.

δi, i – respectively displacement and velocity of the center of mass of wheels front and rear suspension, while driving along a given profile path.

After substitution and solutions in the software product "Wolfram Mathematica" expressions for the kinetic and dynamic characteristics potential energy dissipation function, taking into account dependences (1, 2, 3), and the related transformations we obtain the system of 8 differential equations of second order. The solution of equations describe the changes of generalized coordinates that allow us to estimate the dynamics of the car, taking into account the nature of the road.

The excerpt of the model file is provided below:
ClearSystemCache
Remove["Global'*"]
SetSystemOptions ["BooleanComputation-Options"→Automatic]
Attributes [Attributes]
(*IMPRINT*)
(*Math Model Variable Identifiers*)
(*Mass-inertial characteristics*)
(*Extend the range of the area in the most imperfect zones*)
mk; (*Body weight*)
mp; (*Wheel weight»)
Jy; (*Moment of inertia of the trolley body relative to the Y-axis*)
Jb; (*Moment of inertia of the trolley body relative to the Z-axis*)
Jx; (*Moment of inertia of the trolley body relative to the X-axis*)
Jls ; (*Moment of inertia of the trolley wheel relative to the Z-axis*)
Jly; (*Moment of inertia of the trolley wheel relative to the Y-axis*)

Jlξ=Jlz; (*Moment of inertia of the trolley wheel relative to the Z-axis*))
(*Stiffness and elastic-viscous supports*)
β; C; (*Radial stiffness and RES dissipation value of trolley undercarriage*)
βξ; cξ; (*Radial stiffness and RES dissipation value of the trolley undercarriage*)
βp; cp; (*Horizontal transverse elastic-viscosity resistance m Rail track stiffness*)
β1; C1; (**Elastic-Viscous Resistance and Path Stiffness)
μf (*Coefficient of Friction*);
mm1; (modulus of elasticity of the material 1*)
mm2; (modulus of elasticity of the material 2*)
E2; (*Material Poisson's Ratio 1*)
E1; (*Material Poisson's Ratio 2*)
V0; (*Movement Speed*)
(*Geometric characteristics*)
Dw; (*Ball Diameter*);
α; (*Taper angle of thrust guide and thrust bushings*);
l; (*wheelbase*);
r; (*wheel radius*);
hm; (*height of the center of mass*);
b; (*Trolley Track Width*);
a; (*Track gauge*);
DAngle; (*Limit of angular displacement of the oncoming wheel, rad.*)
cw; (*Wheel flange height*);
δ; (*taper angle of the wheel flange of the mine trolley wheel*);
Skk; (* Rail Track Width Value *);
(* Duration of processes *)

Tmax; (*Model Time*)
(*Wheel Angular Oscillation Ranges*)
Import ["D\Irreg.xls"];
Import ["D\Irreg_Lateral.xls"];
Import ["D\Ri_2.xls"];
(*COMPILATION OF KOENIG'S EQUATIONS*)

$T=1/2$ {mk s'(t)2 +mp [(sl' (t)2 + s2' (t) 2 + s2' (t) 2 + s3' (t) 2 + s4' (t) 2] + [y1' (t)2 + y2' (t)2 + y3' (t)2 + y4' (t)2]} + Jy [(φ' (t)]2 + Jx [(ψ' (t)]2 + {4mp+4Jly/r2} x'(t)2+ξ'(t)2Jlξ+Jx x'(t)2/Ri2};

maximum permissible torque, at which there will be no grip disruption, will be defined from the expression of after substitution the relative velocity. Using the relation between torque and angular velocity of tractive motor, we can determine the voltage as a function of the speed V for these conditions and formulate requirements for tractive motor control algorithm. (**Fig. 4**).

A mathematical model of the motion of dump truck, should be obtained according to evaluated effect on the main road profile and dynamic performance of the trucks.

According to the results of the calculation of plots in **Fig. 5 - Fig. 11** can estimate the shift of front suspensions and the rear wheels, as well as the center of mass of the body. Furthermore, these corresponding values of acceleration, which allows to estimate the inertial load on the parts of the machine design to overcome obstacles.


Fig. 3. Control algorithm

The generalized mathematical model requires times less computational resources using Wolfram Mathematica. As a result of the simulation according to the algorithm (pic. 4, for instance) several engineering results can be obtained.

For mining locomotive it could be the critical velocity by equating the limit values for traction grip and power to each other. The

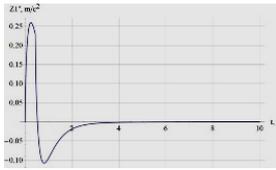*Fig. 4.* Move the center of mass of the front axle dump it, vertically.

Fig. 5. Moving the center of mass rear dump axis and vertically.
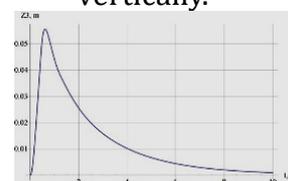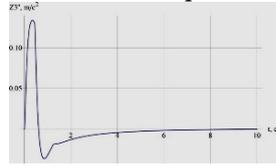

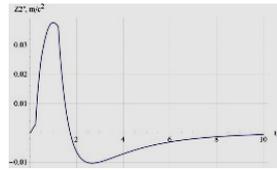*Fig. 6.* Move the center of mass of car-truck vertically.
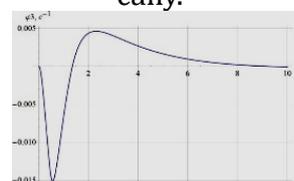
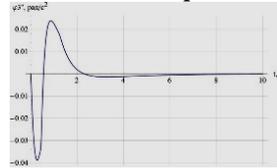*Fig. 7.* The angular displacement relative to the axis of the center of mass of the dump (a longitudinal pitch).

Fig. 8. The vertical acceleration of the front axle dump truck

Fig. 9. The vertical acceleration of the rear axle dump truck

Fig. 10. The vertical acceleration of the center of mass dump

Fig. 11. The angular acceleration relative to the axis of the center of mass dump.

The resulting mathematical model of the motion of career dump, will evaluate the impact of an isolated road irregularities, and structural parameters of the dump on the main dynamic performance and traction. It can be seen as manifested in overcoming obstacles along the way with a constant longitudinal gradient of the road pit dump their own variations.

Conclusions

Computer simulation while analysis of mining vehicles dynamics lets provide research regarding the mechanical design, definition of rational parameters of a new vehicle or modernization of the existing. The usage of applied mathematical model allows developing customized models, which characteristics assessed through known characteristics. Results can be presented as mechanical construction tension, velocities, acceleration of links in order to evaluate reliability and lifetime of machine subject to specific exploitation conditions meanwhile reducing development time.

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